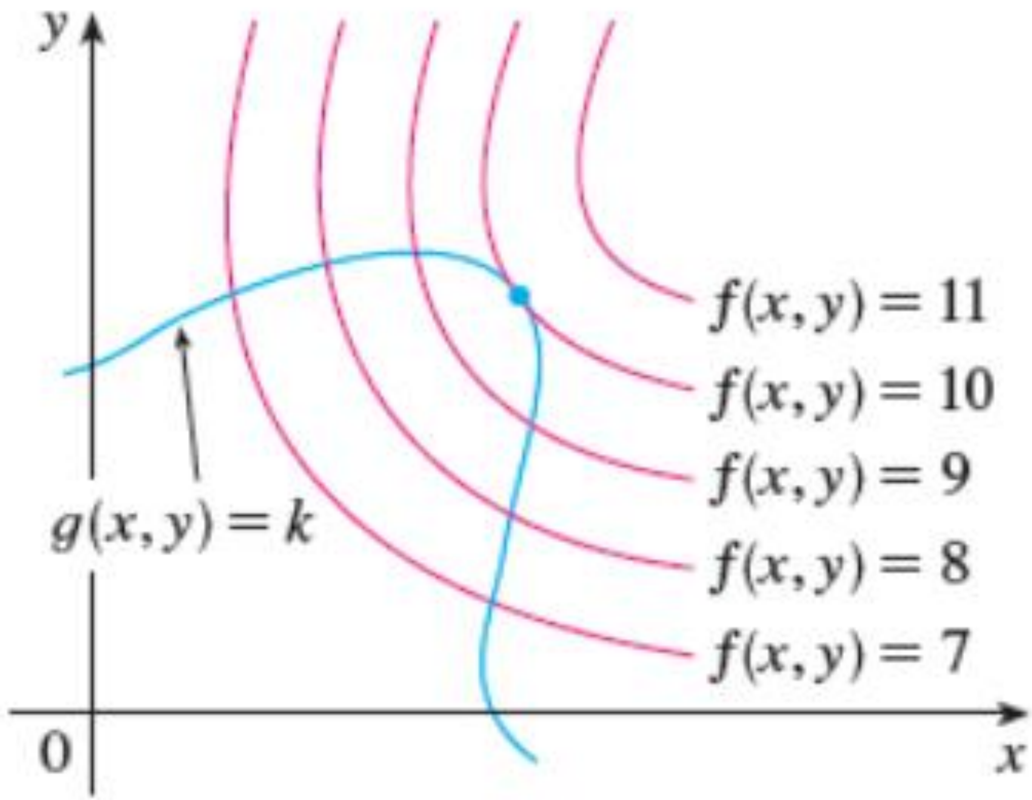
Section 14.8:
Lagrange Multipliers

What We'll Learn In Section 14.8…

1. Method of Lagrange Multipliers (2 variable function, 1 constraint)
2. Method of Lagrange Multipliers (3 variable function, 1 constraint)
3. Method of Lagrange Multipliers (3 variable function, 2 constraints)

1. Method of Lagrange Multipliers (2 variable function, 1 constraint)



<u>Proof</u>:?

1. Method of Lagrange Multipliers (2 variable function, 1 constraint)

<u>Method of Lagrange Multipliers</u>

To find the absolute max and min values of $f(x, y, z)$ subject to the constraint $g(x, y, z) = k$ [assuming that these extreme values exist and $\nabla g \neq \vec{0}$ on the surface $g(x, y, z) = k$]:

a) Find all values of $x, y, z,$ and λ such that

$$\nabla f(x, y, z) = \lambda \nabla g(x, y, z) \quad \text{and} \quad g(x, y, z) = k$$

b) Evaluate f at all the points (x, y, z) that result from step (a). The largest of these values is the maximum value of f; the smallest is the minimum value of f.

1. Method of Lagrange Multipliers (2 variable function, 1 constraint)

Ex 1 (book ex. 2): Find the extreme values of the function $f(x, y) = x^2 + 2y^2$ on the circle $x^2 + y^2 = 1$.

1. Method of Lagrange Multipliers (2 variable function, 1 constraint)

Ex 2 (book ex. 3): Find the extreme values of the function $f(x, y) = x^2 + 2y^2$ on the disk $x^2 + y^2 \leq 1$.

2. Method of Lagrange Multipliers (3 variable function, 1 constraint)

Ex 3 (book ex. 1): A rectangular box without a lid is to be made from 12 m^3 of cardboard. Find the maximum volume of such a box.

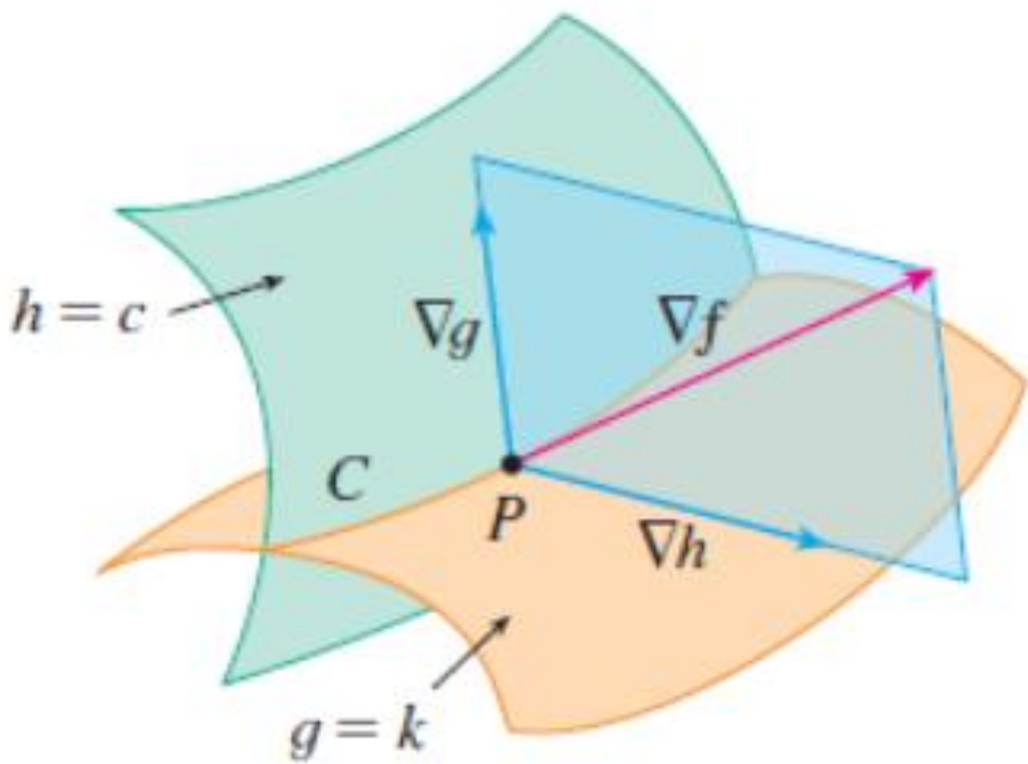
2. Method of Lagrange Multipliers (3 variable function, 1 constraint)

<u>Ex 4 (book ex. 4)</u>: Find the points on the sphere $x^2 + y^2 + z^2 = 4$ that are closest to and farthest from the point $(3, 1, -1)$.

1. Method of Lagrange Multipliers (3 variable function, 2 constraints)

Method of Lagrange Multipliers

$$\nabla f(x,y,z) = \lambda \nabla g(x,y,z) + \mu \nabla h(x,y,z)$$



3. Method of Lagrange Multipliers (3 variable function, 2 constraints)

Ex 5 (book ex. 5): Find the maximum value of the function $f(x, y, z) = x + 2y + 3z$ on the curve of intersection of the plane $x - y + z = 1$ and the cylinder $x^2 + y^2 = 1$.

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<u>Ex 5 (book ex. 5)</u>: Find the maximum value of the function $f(x, y, z) = x + 2y + 3z$ on the curve of intersection of the plane $x - y + z = 1$ and the cylinder $x^2 + y^2 = 1$.